$$u(r,t) = u_o Q + \sum_{m=1}^{\infty} A_m j_1(k_m r) \cos \omega_m t \quad , \tag{30}$$

$$Q = \frac{a}{N\pi} j_1\left(\frac{N\pi r}{a}\right) \pm \frac{4\mu}{3\lambda+2\mu} \frac{r}{N^2\pi^2} \quad , \quad N = \begin{cases} 1,3,5\ldots \\ 0,2,4\ldots \end{cases} \tag{31}$$

The radial stress can be deduced from the displacement solution using [15]

$$\sigma_r(r,t) = (\lambda+2\mu) \frac{\partial u}{\partial r} + 2\lambda \frac{u}{r} - \gamma v \tag{32}$$

Therefore, we have, by substituting equations (6) and (30) into equation (32),

$$\sigma_r(r,t) = 4\mu\, u_o\, S + \sum_{m=1}^{\infty} A_m k_m M_m \cos \omega_m t \tag{33}$$

$$S = \pm \left(\frac{1}{N\pi}\right)^2 - j_1\left(\frac{N\pi r}{a}\right) / \left(\frac{N\pi r}{a}\right) \quad , \quad N = \begin{cases} 1,3,5\ldots \\ 0,2,4\ldots \end{cases} \tag{34}$$

$$M_m = [(\lambda+2\mu) j_o(k_m r) - 4\mu j_1(k_m r)/(k_m r)] \tag{35}$$

Square Pulse

We now can obtain the displacement and radial stress for a square pulse by applying Duhamel's theorem [16] to the solutions expressed by equations (31) and (33). That is

$$u(r,t) = \frac{\partial}{\partial t} \int_o^t F_t(t-t')u'(r,t')\, dt' \tag{36}$$

where $u'(r,t)$ is the solution given by equation (30) for the case of a sudden application of microwave power. An equivalent expression can, of course, be written for the radial stress. Therefore, by substituting equations (12) and (30) into equation (36), we have for the displacement

$$u(r,t) = u_o\, Qt + \sum_{m=1}^{\infty} A_m j_1(k_m r) \frac{\sin \omega_m t}{\omega_m} \quad , \quad o \le t \le t_o \tag{37}$$

$$u(r,t) = u_o\, Qt_o + \sum_{m=1}^{\infty} A_m j_1(k_m r) \left[\frac{\sin \omega_m t}{\omega_m} - \frac{\sin \omega_m (t-t_o)}{\omega_m}\right], \quad t \ge t_o \tag{38}$$

Similarly, we have for the radial stress

$$\sigma_r(r,t) = 4\mu u_o St + \sum_{m=1}^{\infty} A_m k_m M_m \frac{\sin \omega_m t}{\omega_m} \quad , \quad o \le t \le t_o \tag{39}$$

$$\sigma_r(r,t) = 4\mu u_o St_o + \sum_{m=1}^{\infty} A_m k_m M_m \left[\frac{\sin \omega_m t}{\omega_m} - \frac{\sin \omega_m (t-t_o)}{\omega_m}\right], \quad t \ge t_o \tag{40}$$